SEQUENCES AND SERIES — *Answers*

1 **a** 9, 13, 17, 21, 25 **b** 4, 9, 16, 25, 36 **c** 2, 4, 8, 16, 32 **d** $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$

e –1, 4, 21, 56, 115 **f** $\frac{2}{3}, \frac{1}{3}, 0, -\frac{1}{3}, -\frac{2}{3}$ **g** $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}$ **h** 16, 8, 4, 2, 1

2 **a** $u_n = 3n + 1$, $a = 3, b = 1$ **b** $u_n = 7n - 7$, $a = 7, b = -7$ **c** $u_n = 18 - 2n$, $a = -2, b = 18$

d $u_n = 1.3n - 0.9$, $a = 1.3, b = -0.9$ **e** $u_n = 117 - 17n$, $a = -17, b = 117$ **f** $u_n = 8n - 21$, $a = 8, b = -21$

3 possible answers are

a $5n - 4$ **b** 3^n **c** $2n^2$

d $\frac{1}{4} \times 2^n$ **e** $33 - 11n$ **f** $(n - 1)^3$

g $n^2 + 3$ **h** $\frac{n}{2n+1}$ **i** $2^n - 1$

4 **a** $u_3 = c + 3 = 11 \quad \therefore c = 8$

b $u_6 = 8 + 3^4 = 89$

5 **a** $u_4 = 4(8 + k) = 32 + 4k$
$u_6 = 6(12 + k) = 72 + 6k$
$\therefore 72 + 6k = 2(32 + 4k) - 2$
$72 + 6k = 62 + 8k$
$k = 5$

b $u_n = n(2n + 5) = 2n^2 + 5n$
$u_{n-1} = (n - 1)[2(n - 1) + 5] = (n - 1)(2n + 3) = 2n^2 + n - 3$
$\therefore u_n - u_{n-1} = (2n^2 + 5n) - (2n^2 + n - 3) = 4n + 3$

6 **a** $u_1 = k - 3$
$u_2 = k^2 - 3$
$\therefore k - 3 + k^2 - 3 = 0$
$k^2 + k - 6 = 0$
$(k + 3)(k - 2) = 0$
$k = -3$ or 2

b $k = -3 \Rightarrow u_5 = (-3)^5 - 3 = -243 - 3 = -246$
$k = 2 \Rightarrow u_5 = 2^5 - 3 = 32 - 3 = 29$

7 **a** 3, 7, 11, 15 **b** 2, 7, 22, 67

c –2, 1, 7, 19 **d** 5, 2, 5, 2

e –1, 14, –46, 194 **f** 10, 3, 2.3, 2.23

g 6, –1, $1\frac{1}{3}, \frac{5}{9}$ **h** 0, $\frac{1}{2}, \frac{2}{5}, \frac{5}{12}$

8 possible answers are

a $u_{n+1} = u_n + 4,\ u_1 = 5$ **b** $u_{n+1} = 3u_n,\ u_1 = 1$ **c** $u_{n+1} = u_n - 18,\ u_1 = 62$

d $u_{n+1} = \frac{1}{2}u_n,\ u_1 = 120$ **e** $u_{n+1} = 2u_n + 1,\ u_1 = 4$ **f** $u_{n+1} = 4u_n - 1,\ u_1 = 1$

9 **a** $-3 = -4a + b$
$-1 = -3a + b$
subtracting, $2 = a$
$a = 2, b = 5$

b $8 = b$
$4 = 8a + b$
$a = -\frac{1}{2}, b = 8$

c $4 = \frac{11}{2}a + b$
$3 = 4a + b$
subtracting, $1 = \frac{3}{2}a$
$a = \frac{2}{3}, b = \frac{1}{3}$

10 **a** $u_2 = 4 + 3k$
$u_3 = 4(4 + 3k) + 3k = 16 + 15k$

b $u_2 = 2k + 5$
$u_3 = k(2k + 5) + 5 = 2k^2 + 5k + 5$

c $u_2 = 4k - k = 3k$
$u_3 = 4(3k) - k = 11k$

d $u_2 = 2 + k$
$u_3 = 2 - k(2 + k) = 2 - 2k - k^2$

e $u_2 = \frac{4}{k}$
$u_3 = \frac{4}{k} \div k = \frac{4}{k^2}$

f $u_2 = \sqrt[3]{61k^3 + 3k^3} = \sqrt[3]{64k^3} = 4k$
$u_3 = \sqrt[3]{61k^3 + 64k^3} = \sqrt[3]{125k^3} = 5k$

11 **a** $u_2 = \frac{1}{2}(k + 6)$
$u_3 = \frac{1}{2}[k + \frac{3}{2}(k + 6)] = \frac{1}{4}(5k + 18)$

b $\frac{1}{4}(5k + 18) = 7$
$k = 2$
$u_4 = \frac{1}{2}(2 + 21) = 11\frac{1}{2}$

12 **a** $u_4 = 30 - 2 = 28$
$10 = 3u_2 - 2 \;\therefore\; u_2 = 4$
$4 = 3u_1 - 2 \;\therefore\; u_1 = 2$

b $u_4 = \frac{15}{4} + 2 = 5\frac{3}{4}$
$5 = \frac{3}{4}u_2 + 2 \;\therefore\; u_2 = 4$
$4 = \frac{3}{4}u_1 + 2 \;\therefore\; u_1 = 2\frac{2}{3}$

c $u_4 = 0.2 \times 1.2 = 0.24$
$-0.2 = 0.2(1 - u_2) \;\therefore\; u_2 = 2$
$2 = 0.2(1 - u_1) \;\therefore\; u_1 = -9$

d $u_4 = \frac{1}{2}$
$1 = \frac{1}{2}\sqrt{u_2} \;\therefore\; u_2 = 4$
$4 = \frac{1}{2}\sqrt{u_1} \;\therefore\; u_1 = 64$

13 **a** $u_5 = 2 + 4c = 30 \;\therefore\; c = 7$
b sequence is 2, 9, 16, 23, 30, …
$\therefore\; u_n = 7n - 5$

14 **a** $u_2 = 3(-4 - k) = -12 - 3k$
$u_3 = 3[(-12 - 3k) - k] = -36 - 12k$
b $-36 - 12k = 7(-12 - 3k) + 3$
$9k = -45$
$k = -5$
c $u_3 = -36 + 60 = 24$
$\therefore\; u_4 = 3(24 + 5) = 87$

15 **a** $t_2 = 1.5k + 2$
$t_3 = k(1.5k + 2) + 2 = 1.5k^2 + 2k + 2$
b $1.5k^2 + 2k + 2 = 12$
$3k^2 + 4k - 20 = 0$
$(3k + 10)(k - 2) = 0$
$k = -3\frac{1}{3}, 2$